

MAPPING A PRICING PROCESS THROUGH A FUZZY INFERENCE SYSTEM: DECISION-SUPPORT FOR A SMALL BUSINESS ENTREPRENEUR

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ABSTRACT

Many business owners face struggles in determining their prices even with great experience and knowledge in their fields. The present paper addresses one case of this issue: a professional and entrepreneur that offers mechanical and lathe services in his own workshop. The goal of this work was to map the intuitive pricing process of the entrepreneur through a Fuzzy Inference System (FIS). Many fuzzy aspects such as the imprecision, uncertainty, and ambiguity of these lathe and maintenance services were related to the potential benefits of FIS, clarifying which methods were used and why. This FIS was constructed to mimic the empirical pricing process of the referred professional, and, in this task, this project was successful. Output surfaces showed that complexity and goods value have significant effects just above medium levels and that their impact has a smaller weight than the estimated time. Furthermore, some unexpected outcomes were reached in the system development. Not only was the entrepreneur's reasoning mapped but it also provided more understanding of his own market.

Keywords: Fuzzy Logic; Pricing; Fuzzy Inference System; Small Business.

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1 INTRODUCTION

Many business owners face struggles in determining their prices even with great experience and knowledge in their fields. There are many references and consolidated theories regarding pricing strategies depending upon factors such as target audience, location, market share, new competitors, and so on. In some cases, the easiest methods are just mimicking the values charged by their competitors for similar products or services or present prices slightly under this level. However, it might be difficult in a market that is characterized by a huge diversification and scarce available information.

The present paper addresses one case of this issue: a professional and entrepreneur that offers mechanical and lathe services in his own workshop. Besides the difficulty to identify similar services in the market, he also deals with demands that request a variety of competencies (machining, welding, industrial drawing, and others) applied on several applications such as cars, boats, trucks, bicycles, and any metal parts. The professional spends a lot of time developing technical skills and has field experience, obtained during the years working for international offshore companies. However, he does not have any managerial training and, as an entrepreneur, has many struggles to assess the fair prices of his services. The difficulties in performing this task derive not only due to his low administrative skills but also from its complexity. How many variables are related to generating a suggested price and how?

The goal of this work was to map the intuitive pricing process of the entrepreneur through a Fuzzy Inference System (FIS). It could enable him to identify which factors are relevant to him in the pricing process - the input variables - and also how they are related between themselves to estimate the fair price - the rulebase. Once the variables involved in this procedure are not always clearly defined, a Fuzzy Inference System (FIS) was selected as a method to support and map this decision-making process, since it can deal with ambiguity and also perform nonlinear computations with both numbers and words. The applications of this method are increasingly growing and, in the Knowledge-Based Systems area, fuzzy logic and decision-making are among the six main research areas (Zhang et al., 2017).

Humans often use classification to perform analysis. Although very useful, often these built classes are imprecise, not showing a clear and objective distinction between similar classes. Even, in broader classes, it might not ensure within its components the same distinctive characteristic. In his seminal paper, Zadeh (Zadeh,1965) already highlighted the inherent imprecision in the "classes" definitions in human thinking and how it was very relevant in areas such as abstraction, pattern recognition, and communication. Even with the computational and informational advances - starting from time with much less available data and small processing capacity to time with big data, data mining and so on - the imprecision about the defined "classes" is still a critical aspect (Meyer & Zimmermann, 2011).

This way, fuzzy logic can cover some spots where traditional control or statistics tools are not sufficient to obtain significant results, dealing well with the lack of standardized data (Garcia & Velasquez,2013; Ramirez,2010). The use of Fuzzy Logic can be a useful way to map input space to output space, with uses on control, predictions, and evaluation systems. To apply Fuzzy Logic in real problems, there are deduction systems that convert input data to output variables, generating some control or prediction such as Fuzzy Inference Systems. These properties are very useful since small enterprises have managerial limitations that demand user-friendly systems, easy to comprehend and operate.

The next section introduces the fuzzy set theory including its practical applications through inference systems. The third section explains the study case methodology and the research tools used to collect data, map the critical variables and identify relationships to build the rule base. Also, many fuzzy aspects such as the imprecision, uncertainty, and ambiguity of

these lathe and maintenance services were related to the potential benefits of FIS, clarifying which methods were used and why. After that, section five presents the inference system itself: variables and fuzzy sets, surface outputs, and the two-stage design, describing its properties. At least, the conclusions of this work are presented, highlighting its limitations and possible further developments. Besides that, some additional benefits of the building process to the entrepreneur are also reported.

2 FUZZY LOGIC AND FUZZY INFERENCE SYSTEMS

Fuzzy Logic deals with the belongingness of objects in a set, defined by a degree called membership function. Every element should be a tuple x (the object itself) and μ_x (the degree of membership in the concept of the set). This value can vary from 0 - absolute nonbelongingness - up to 1 - full adequacy to the concept of set A . In its extreme cases, classical (hard, crisp, non-fuzzy) sets can be considered a restricted case of the more general concept of fuzzy set (Bodjanova,2000).

The technological and scientific advances based on conventional logic are undeniable, but some limitations can bring some interest in the use of Fuzzy logic (Zadeh,1973). First, as stated before, Fuzzy deals with imprecise or vague concepts, characteristics of many categorization processes. Second, its inference can deal with nonlinear systems without the need for linearization of this environment, turning it more adherent to the real environment. At least, can be based on human-centric thinking, causing it to be more intuitive and faster to construct. As it deals well with linguistic variables, it's easier to transcribe human experiences to construct both the fuzzy sets and the rule base, which in its canonical form (If x is A_1 then y is B_1) is more usual to the human thinking than functions modeling with many non-linear variables.

Fuzzy Logic, since it is multi-valued, can surpass this dichotomy dealing with imprecise or vague data. These values are belongingness degrees to the elements of a specific set or, in other words, degrees of truth of some statement (Garcia & Velasquez,2013; Ramirez,2010). This capacity is useful for Social Sciences since even the symbols are not fully precise and its questions have, frequently, a great number of influencing factors. In the real world, except for very simple statements (odds or even numbers) there is nothing black-white, hot-cold. Every classification - or every concept - does not possess sharp boundaries (Pedrycz & Gomide,2007). And to apply this concept in many cases, including decision-making processes, there is useful the construction of Fuzzy Inference Systems.

To apply Fuzzy Logic in real problems there are deduction systems that convert input data to output variables generating some control or prediction. The overall procedure can be seen in Figure 1 and generally is composed of fuzzification, inference, aggregation, and defuzzification (Pedrycz & Gomide,2007; Herrera et al., 2002; Andrade & Jacques, 2008; Kaur & Kaur,2012). The first step of the system is fuzzification. In this part, membership functions are assigned for each input variable, even if this data is fuzzy or a crisp (non-fuzzy) number.

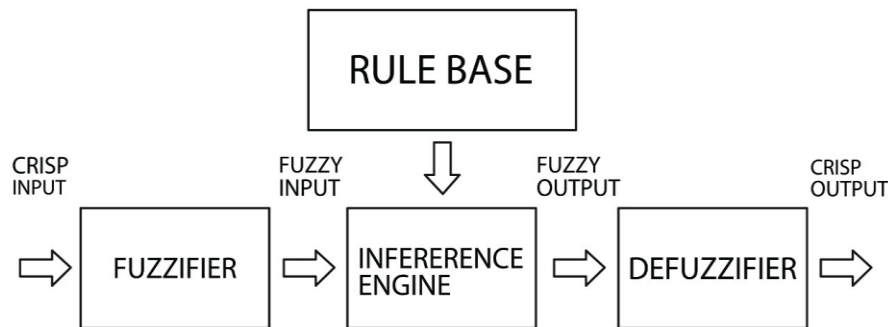
The next step is the inference based on a predefined rule database. In this paper, as will be explained further, we use the Mamdani method. But, anyway, the two main inference types – Mamdani or Sugeno - uses rules in the same canonical forms "If x_1 is A_1 then y_1 is B_1 ". The statement above is with just one input and one outcome. The capital letters are the sets, x the input and y the output value. Also, rules can be composed of as many inputs or output variables as it needs, and their general forms are described as follows:

If x_1 is A_1 (conector) x_2 is A_2 (conector) ... X_n is A then y_1 is B_1 (conector) y_2 is B_2 (conector) ... Y_n is B_n

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These implication rules are composed also by connectors as "and" or "or", which implies disjunction or conjunction operations used to generate the system responses. Each fired rule generates an output fuzzy set with an activated membership degree, depending upon the input values and the connector used to compose the rules. However, this output is for each rule and not for the system at all. This way, many responses can be identified, and a further step is necessary to create a single output (for each variable).

Figure 1- Fuzzy Inference System



Source: Author elaboration.

The aggregation step combines several rules outputs to generate a single fuzzy set for each system output variable. Is the decision of whom is modeling the system to choose between operators and methods for aggregation, which can also change the obtained values. Depending upon the application the process can stop here but, in general, there is a need to transform this fuzzy set into a crisp number to be used in the control. Also, this time, there are many defuzzification methods - centroid, minimum of the maximum, middle of maximum, e.g. - that generates different responses and also gets their benefits and weaknesses.

The FIS scope is to support the pricing process of customized mechanical and lathe services for a small business entrepreneur. To best fit the decision support tool to the case, it was necessary to better understand the possibilities and limitations of the final user regarding his managerial knowledge and also the empirical method used to price definition

3 RULE BASE CONSTRUCTION METHODOLOGY

The first stage was composed of unstructured interviews and documentary analysis. The interviews were useful to identify the main variables of the pricing process and map the pricing reasoning of the professional through rules. All interviews were conducted with the entrepreneur, responsible not only for managing the workshop but also to perform the majority of the operational tasks. The first one was directed to the identification of the variables, both inputs and outputs, and a preliminary fuzzy sets mapping. Between the two interviews, a data survey was conducted collecting data from forty services, registering their parameters in each input and output variable. Also, operational descriptions of each observation were collected to provide information for the core/border data anchors. With these data, a first sketch of the FIS was built. The second interview discussed the preliminary structure and the relationship

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between variables. The focus was to build the rule base and to identify the conceptual characteristics of each border/core definition for each fuzzy set within each variable.

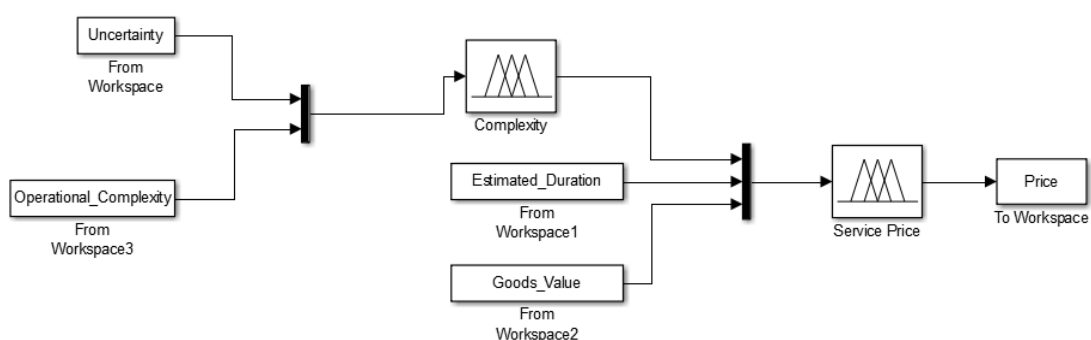
Membership functions depend on the uncertainty degree of the assessments and the judgment granularity, and the fuzzy set support (its range) were determined by the fuzziness of the variable. In the previous step were defined four variables and aimed to reduce the complexity of the system, there were chosen three membership functions per variable. This choice was also supported because of the risks of harming the system's quality. The construction process of a FIS through mapping of human-based knowledge is highly prescriptive as it tries to transcribe an expert's domain knowledge. So, it was necessary to evaluate the completeness and consistency - two measurements of FIS quality - of the rule database.

Completeness means that every input value has a membership function bigger than zero in at least one fuzzy set. And since every input fuzzy set is linked to an outcome, it means that every input can be used as a condition to fire the rules with nonzero weight and thus generate a response (Gonzalez & Perez, 1998). Additionally, the fuzzy sets shapes were triangular or trapezoidal to avoid the need for rule base significance thresholds - establishing a minimum membership value to consider the rule as activated. This restriction would be useful for membership shapes that still activate outputs in very low values, such as Gaussian curves.

To ensure completeness of this works a rules matrix was built, generating a nonlinear mapping of the original spaces, including every possible rule for the system (Pedrycz & Gomide, 2007)). However, it generates problems in another quality measurement of the rule base, which is consistency. Consistency means that similar rules cannot generate completely different or contradictory rules (Gonzalez & Perez, 1998). These problems might occur when there is an increasing number of variables. For instance, if we use four inputs with just three membership functions, the rule database is fully covered with sixty-four rules. If we use five inputs and the same number of membership functions within these variables, there will be necessary one hundred twenty-five rules to ensure the system's completeness.

Besides the weakness that a complete rulebase demands more processing, combining every possible rule can generate inconsistent conclusions. To improve system consistency, we will use the method of rules combination [x]. Diagnoses can indicate the composing of one rule mixing two rules, forming new "any" rules to consolidate several rules, and combining combined rules. This process is complementarily developed with the entrepreneur revision of the system until appropriate results are reached. At least, a calibration framework was applied through the definition of data anchors for each fuzzy set (Legewie, 2017).

Figure 2 - Block Diagram of the two-stage Fuzzy Inference System in Simulink



Source: Author elaboration.

The FIS was built on the software Matlab which has the Fuzzy Logic Toolbox - useful to build the inference systems. This resource presents a graphic interface that allows the inclusion/exclusion of variables, construction, calibration of membership functions, and the choice of operators, aggregation and defuzzification methods. Also, the simulation platform Simulink was used to create the two-stage FIS, allowing communication with other software. Simulink is a Matlab feature for modeling, simulation, and analyzing dynamic systems with graphic block diagrams (Mathworks,2016).

3 FUZZY INFERENCE SYSTEM DEVELOPMENT

The pricing process is performed by a two-stage inference system as can be seen in Figure 2. The final price is an output of a FIS which has the complexity of the task, the estimated time to be spent on the maintenance/construction, and the value of the goods. In its turn, the complexity was decomposed into planning uncertainty and the operational complexity of the service. The fuzzy sets classical examples were defined through a concept tree, core and border conceptual definitions, and data gathering of iconic observations for each of these components. The overall process follows the anchored calibration method proposed by (Legewie,2017).

Complexity was composed of two other variables that through a FIS translated into five membership functions. This process decomposition was done through a formulation of a concept tree, determining relevant variation and establishing archetypical characteristics of each set (Legewie,2017). This was useful to break the abstract notion of Complexity into two indicator-level inputs that were staggered and more useful to the entrepreneur assessment of the hired services.

The first input for Complexity is Uncertainty, which represents the predictability degree of the hired task. For instance, foreseeing what problems might occur when pricing welding services in older cars is a little bit difficult. They can be rusted in some spots that would demand other interventions. Or the owner or the former ones may change some original parts, thus difficult for the professional to identify potential risks. On the other hand, welding services in new cars are less unpredictable in terms of what can and cannot happen during the task. However, as stated before, the anchored calibration of this variable is not thoroughly explained in this paper.

Operational Complexity was measured by the need for dexterity, technical ability, and specific equipment - higher requirements represent higher operational complexity. This indicator-level dimension was detailed covering its full conceptual continuum through the mapping of qualitative data pieces that represents the fuzzy set definition. It involves not only determining the core characteristics but also the border ones as can be seen in Table 1. After that, data anchors were defined based on actual data from the executed services sample set.

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Table 1- Core and border for indicator-level dimension "Operational Complexity"

Membership Function	Characteristic		Data Anchor
Low-Skill	Core	simple straight cuts of isolated parts equipment with no consumables demand	cutting a 3 meters iron bar in 6 pieces of 0.50 meters each for a gate construction
	Upper Border	reassemble parts risk of damaging a non-critical part of the good	exchange a bearing of a washing machine
Routine	Lower Border	equipment with consumables demand	circular saw cuts on 15 millimeters steel plates
	Core	cutting shapes on isolated parts demands industrial equipment	axis cut and centralization
	Upper Border	cuts on embodied parts risk of damaging a critical part of the good	motorcycle customization
High Complexity	Lower Border	involve precision cuts demands technical drawing skills	off-road automotive owning design and construction
	Core	demands specialized machinery and skills Evident risk of goods loss	industrial cement mixer maintenance

Source: Author elaboration.

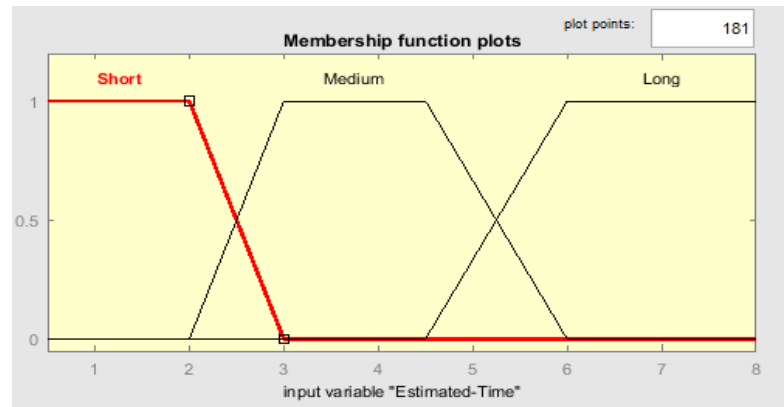
In fuzzy sets, the membership degree represents the belonging of a given element to the classic concept defined for the fuzzy set. If its value is equal to one, it means that the element has a full adherence to the criteria that distinguish the fuzzy partition of others in the same universe of discourse. Triangular membership functions determine that just one point in X full belongs to each membership curve. Trapezoidal curves can have two or infinite - if the domain is continuous - elements with $\mu_x = 1$ since they are circumscribed between the parameters m and n.

Given these characteristics, some questions should be done about the variable Estimated Time and its trapezoidal membership functions. Of course, when the service is done, we have

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a clear number that indicates the time spent doing so. There is no relevant fuzziness in this case since there is very little imprecision, ambiguity, or uncertainty.

Figure 3. Second Stage Input Variable Estimated-Time



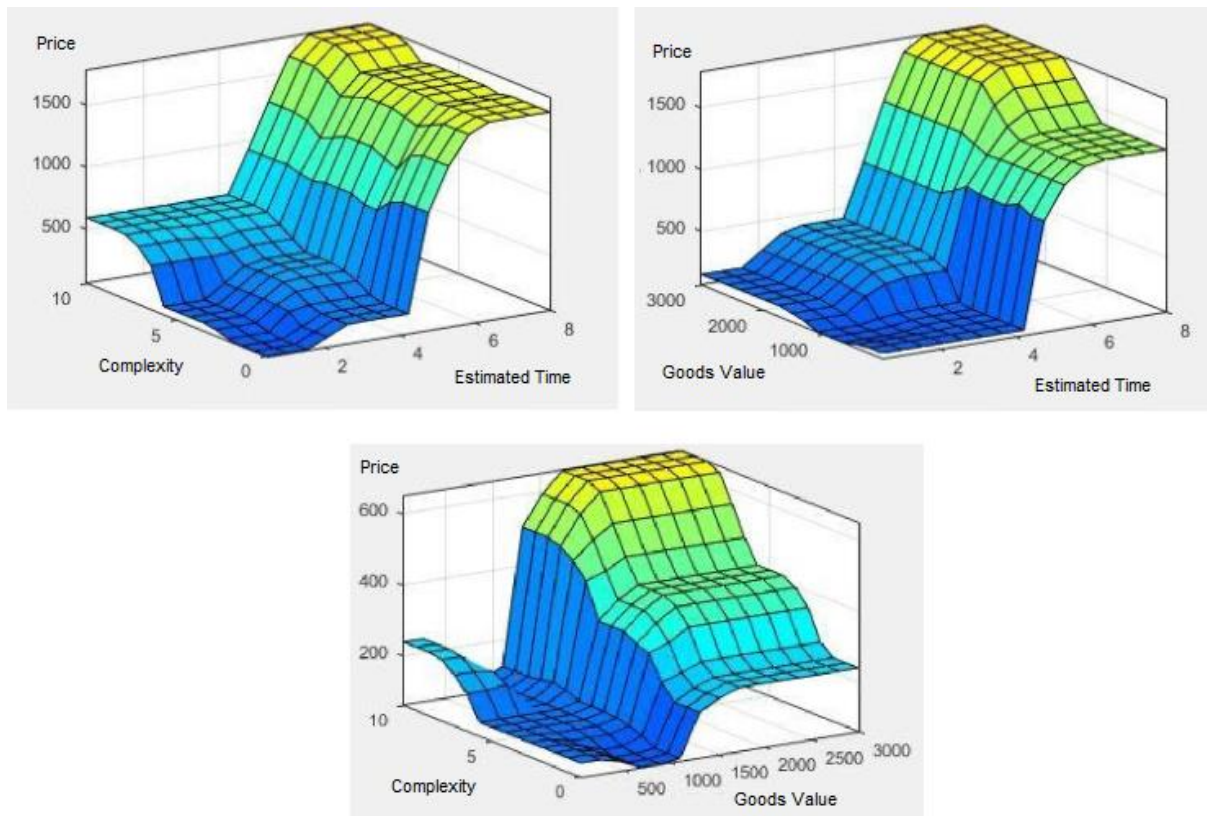
Source: Author elaboration.

However, we should keep in mind that the price is established before the agreement about the effective service. So, the professional has an imprecise and uncertain forecast about the time that will be demanded to finish the job. Factors such as the weather, goods condition, previous goods maintenance, and its effective use might harm the time forecast precision. Also, the professional mood, stamina, and hired services backlog might influence this decision.

In the end, there were applied two distinct methods for determining fuzzy sets, its core concepts, and also borders. For Uncertainty and Operational Complexity (and thus, Complexity), anchored calibration was needed since they are a more imprecise and vaguer concept. Regarding Estimated Time and Goods Value, the inputs are quantitative and discrete. The translation of the entrepreneur beliefs was sufficient to build fuzzy sets and estimate their parameters.

After the rule-based quality assessment, there were generated the output surfaces. Another feature of the fuzzy toolbox in Matlab is that a surface curve is brought forth. For a three-input / one-output system like this one, there are three possible surfaces using the price as an outcome. Each one of these was reviewed with the entrepreneur, clarifying the pricing process for every part involved in the system development - entrepreneur and researchers (Mathworks,2016).

Figure 4. Second Stage Output Surfaces



Source: Author elaboration.

The Estimated Time does not present much relevance on price formation in services that are planned to last less than four hours unless the tasks demand higher levels of Complexity. This reasoning is aligned with the entrepreneurs' policy since the upper complexity levels involve risks of goods loss and also specialized machinery. On the other hand, high-time foresight incurs in upper price levels despite the Complexity of the activity.

A similar pattern can be seen in the plot of Price, Goods Value, and Estimated Time. There are no significant price increases until the medium-duration services are reached. This time, the duration is even more relevant to the price formation. Goods with low values reduce the prices in long-term services, but higher-value goods do not present so significant influence in the pricing of short-term services, in comparison with high complexities.

The major difference is clear in the third image. The surface does not reach half of the range of Price, and it shows that the entrepreneur assigns bigger weight to Estimated Time. Within this reduced range, high-complexity levels lower prices in medium goods values than in lower values. Also, significant price differences are just present above medium core goods values.

This last plot is the most representative of the empirical pricing process of the professional and his difficulty to perceive, as a whole, the relationship between four possible inputs in a non-linear relationship. Although his claims about the need for constant investment for better equipment and consumables, Operational Complexity (and thus Complexity) has a smaller influence on pricing. Regarding Goods Value, it is clear that the price jumps when the goods are valued at more than one thousand Reais, showing a more binary behavior. If it costs

less than that, this variable has a small influence, on the contrary, some amount is added to the price regardless of if it has medium or expensive values.

5 CONCLUSION

The pricing process is not always a clear and fully understood task and the increase of input/output variables turns it exponentially complex. Furthermore, several nonlinearities harm the human comprehension of this issue. In the present paper, we mapped a pricing reasoning of an entrepreneur through a FIS. This work presents a distinct use not only for FIS but also for the system development itself. Seminal applications of fuzzy logic focused on industrial controls converting human knowledge to a controller (Mamdani, 1973; Takagi & Sugeno, 1985). In social sciences, there are applications in several areas such as finance, operations, innovation capabilities assessment and so forth (Garcia & Velasquez, 2013; Ramirez, 2010; Hurtado, 2006; Alfaro et al., 2015; Cherri et al., 2011). The present paper developed a FIS to support a small enterprise and, besides that, clarified the entrepreneur about his intuitive pricing process. The interviews after the first sketch of the FIS, the rule base mapping, and the discussions about the output surfaces provided a better business overview of the workshop owner.

Based on non-structured interviews, direct observation, and documental analysis there were identified three main inputs for this pricing - complexity, estimated time, and goods value. However, during the concept tree development, it was perceived that complexity was better explained through two previous inputs: operational complexity and uncertainty. After the rule base construction, the analysis of the output surfaces allowed the discussion with the entrepreneur, improving his understanding of his empirical pricing process. A set with thirty new observations was tested and the deviations are significantly low from the actual prices (significant here means that the entrepreneur pointed out no relevant monetary difference).

Previously, the entrepreneur assumed that the three second-stage variables had similar weights and a linear relationship. However, the estimated time to the service execution surpasses the relevance of the other inputs. Furthermore, the FIS and its output surfaces showed that complexity and goods value have significant effects just above medium levels and that their impact has a smaller weight than the estimated time, as can be seen in the third surface which does not surpass six hundred reais value, within a range from fifty up to two hundred.

This FIS was constructed to mimic the empirical pricing process of the referred professional and, in this task, this project was successful. Furthermore, some unexpected outcomes were reached in the system development. Not only was the entrepreneur's reasoning mapped but it also provided more understanding of his market. In the end, he decided to focus on business-to-business negotiations since their usual demands are concentrated on higher surface levels. Based on the output surfaces he decided to decrease his focus on individual clients, which were the focused segment, to aim at maintenance companies.

For further FIS developments also some steps proposed by Legewie (Legewie, 2017) should be followed. The services should be documented, scoring each case on indicator-level dimensions. During this process, many empirical observations might be ambiguous and then these specific cases should be more detailed. The last step is to analyze these ambiguous cases to reduce grey areas in the classification process, through rule-base aggregation or parameter refining.

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